

# Cambridge IGCSE<sup>™</sup>

| CANDIDATE<br>NAME |                     |                     |                     |
|-------------------|---------------------|---------------------|---------------------|
| CENTRE<br>NUMBER  |                     | CANDIDATE<br>NUMBER |                     |
| CAMBRIDGE         | INTERNATIONAL MATHE | EMATICS             | 0607/52             |
| Paper 5 Investi   | gation (Core)       |                     | February/March 2021 |
|                   |                     |                     | 1 hour 10 minutes   |

You must answer on the question paper.

No additional materials are needed.

### INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

### INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [].

### **CONSECUTIVE NUMBERS (36 marks)**

This task is about what happens when consecutive numbers are changed from positive to negative and added.

Consecutive numbers are sequences of integers which increase by 1 from term to term.

Examples 0, 1, 2, 3, 4 or 5, 6, 7 or 46, 47, 48, 49, 50, 51 or 3, 4, 5, ..., 120.

In this investigation use this **method** throughout.

- Add the positive consecutive numbers.
- Find all the possible additions and totals when you make **one** of the numbers negative.
- Find all the possible additions and totals when you make **two** of the numbers negative.
- Continue in this way until all the numbers are negative.
- 1 1, 2 is a sequence of two consecutive numbers.
  - (a) (i) Complete the table using the method with 1 and 2 to find all the possible totals.

|              | A  | Total |    |   |    |
|--------------|----|-------|----|---|----|
| All positive | 1  | +     | 2  | = | 3  |
| One negative | -1 | +     | 2  | = |    |
| One negative | 1  | +     | -2 | = |    |
| All negative | -1 | +     | -2 | = | -3 |

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(ii) Using the consecutive numbers 1 and 2 the highest total is 3 and the lowest total is -3.

You cannot make all the integers between the highest total and the lowest total using the method.

Write down all the integers between 3 and -3 that **cannot** be made using 1 and 2. Remember: 0 is an integer.

| <b>(b)</b> | (i) | Complete the table using the method with the consecutive numbers 2 and 3. |
|------------|-----|---|
|------------|-----|---|

|              | A  | Total |    |   |   |
|--------------|----|-------|----|---|---|
| All positive | 2  | +     | 3  | = | 5 |
| One negative | -2 | +     |    | = | 1 |
| One negative | 2  | +     |    | = |   |
| All negative | -2 | +     | -3 | = |   |

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(ii) Using the table in **part** (i), complete these statements.

(c) (i) Complete the table using the method with two consecutive numbers.

|              | Ac | Total |       |     |
|--------------|----|-------|-------|-----|
| All positive |    | +     | <br>= | 15  |
| One negative |    | +     | <br>= |     |
| One negative |    | +     | <br>= |     |
| All negative |    | +     | <br>= | -15 |

[2]

(ii) Find the number of integers between 15 and -15 that **cannot** be made using these consecutive numbers.

......[1]

- 2 a and a + 1 are two consecutive numbers.
  - (a) Find expressions for the four totals that can be made using a and a + 1. Give each expression in its simplest form.

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(b) An expression for the number of integers between the highest total and the lowest total that cannot be made using a and a + 1 is 4a - 1.

Show that this gives the correct number when a = 10.

- 3 (a) There are now three consecutive numbers.
  - (i) Complete the table using the method with the consecutive numbers 3, 4 and 5.

| Addition |                        |   |  |  |  | Total  |
|----------|------------------------|---|--|--|--|--|
| 3        | +                      | 4   | +  | 5  | =  | 12   |
| -3       | +                      | 4   | +  | 5  | =  | 6  |
|          | +                      | -4  | +  |  | =  | 4  |
|          | +                      |   | +  |  | =  | 2  |
| -3       | +                      | -4  | +  |  | =  | -2   |
| -3       | +                      |   | +  | -5   | =  |  |
|          | +                      |   | +  |  | =  |  |
| -3       | +                      | -4  | +  | -5   | =  | -12  |
|          | -3<br><br>-3<br>-3<br> | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ |

(ii) Find the number of integers that **cannot** be made between 12 and -12.

https://xtremepape.rs/

[2]

(b) There are now four consecutive numbers.

|                |    |   |    | Add | ition |   |    |   | Total |
|----------------|----|---|----|-----|-------|---|----|---|-------|
| All positive   | 3  | + | 4  | +   | 5     | + | 6  | = | 18    |
|                | -3 | + | 4  | +   | 5     | + | 6  | = | 12    |
| One negative   |    | + | -4 | +   | 5     | + | 6  | = | 10    |
| One negative   | 3  | + | 4  | +   | -5    | + | 6  | = | 8     |
|                |    | + |    | +   |       | + |    | = |       |
|                | -3 | + | -4 | +   | 5     | + | 6  | = |       |
|                | -3 | + | 4  | +   |       | + | 6  | = | 2     |
| Two magative   | -3 | + | 4  | +   | 5     | + | -6 | = | 0     |
| Two negative   | 3  | + | -4 | +   |       | + | 6  | = | 0     |
|                | 3  | + | -4 | +   |       | + | -6 | = |       |
|                | 3  | + | 4  | +   | -5    | + | -6 | = | -4    |
|                | -3 | + | -4 | +   |       | + |    | = |       |
| Three positive | -3 | + | -4 | +   |       | + |    | = | -8    |
| Three negative | -3 | + | 4  | +   | -5    | + |    | = | -10   |
|                | 3  | + |    | +   |       | + |    | = | -12   |
| All negative   | -3 | + | -4 | +   | -5    | + | -6 | = | -18   |

Complete the table using the method and the consecutive numbers 3, 4, 5 and 6.

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https://xtremepape.rs/

# TURN OVER FOR QUESTION 4

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4 (a) There are 16 additions in the table on page 6.

Complete the table below. Use **Question 1** and **Question 3** to help you.

| Number of<br>consecutive<br>numbers | Number o | ions |                |
|-------------------------------------|----------|------|----------------|
| 2                                   |          | =    |                |
| 3                                   |          | =    | 2 <sup>3</sup> |
| 4                                   | 16       | =    | 2 <sup>4</sup> |
| 5                                   | 32       | =    |                |
|                                     |          |      |                |
| n                                   |          |      |                |

(b) Complete this table. Use **Question 2(a)** to help you.

| Number of<br>consecutive<br>numbers | Consecutive numbers           | Expression for the highest total in terms of <i>a</i> |
|-------------------------------------|-------------------------------|---|
| 2                                   | <i>a</i> , <i>a</i> + 1       |   |
| 3                                   |                               | 3a + 3  |
| 4                                   |                               |   |
| 5                                   | a, a + 1, a + 2, a + 3, a + 4 |   |
| <br> <br> <br>                      |                               |   |
| n                                   |                               | $+\frac{n(n-1)}{2}$                                   |

[2]

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- (c) Anna uses this method to work out the number of integers that **cannot** be made.
  - Use **Question 4(b)** to find the highest total.
  - Find the number of integers from the highest total to the lowest total.
  - Use **Question 4(a)** to find the number of additions.
  - Subtract the number of additions from the number of integers.

### Example

There are three consecutive numbers. The first number is 4.

The highest total is  $3a + 3 = 3 \times 4 + 3 = 15$ . The number of integers from 15 to -15 is 31. The number of additions is  $2^3$ . The number of integers that **cannot** be made is  $31 - 2^3 = 23$ .

(i) There are two consecutive numbers.

Use Anna's method to find the number of integers that **cannot** be made when the first number is 9.

.....[3]

(ii) Anna uses her method to find the number of integers that cannot be made with the three consecutive numbers 1, 2 and 3. Her method gives the answer 5.

Explain why her method gives the wrong answer.

## .....[3]

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